# Instruction Selection on SSA Graphs

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Compiler Construction Course
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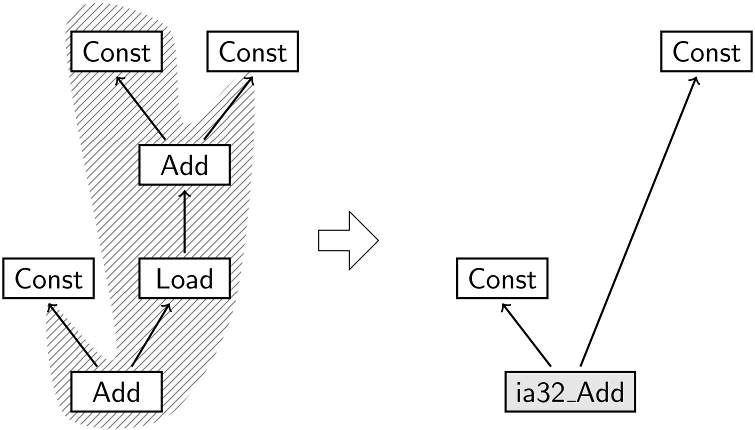
# Instruction Selection



# Instruction Selection on SSA

- "Optimal" instruction selection on trees is polynomial
- SSA programs are directed graphs
  $\implies$ Data dependence graphs
- Translating back from SSA graphs to trees is not satisfactory
- "Optimal" instruction selection is NP-complete on DAGs
- The problem is common subexpressions
- Doing it on graphs provides more opportunities for complex instructions:
  - Patterns with multiple results
  - DAG-like patterns

# Instruction Selection on SSA

- Graph Rewriting
- For every machine instruction specify:
  - A set of graphs (patterns) of IR nodes
  - Every pattern has associated costs

1. Find all matchings of the patterns in the IR graph
2. Pick a correct and optimal matching
3. Replace each pattern by corresponding machine instruction

$\Longrightarrow$ Result is an SSA graph with machine nodes

# Graphs

- Let $G = (V, E)$ be a directed acyclic graph (DAG)
- Let $Op$ be a set of operators
- Every node has a degree $\deg v : V \rightarrow \mathbb{N}_0$
- Every node $v \in V$ has an operator: $\mathrm{op} : V \rightarrow Op$
- Every operator $o \in Op$ has an arity $\# : Op \rightarrow \mathbb{N}_0$
- Let $\square \in Op$ be an operator with $\# \square = 0$
- Nodes with operator $\square$ denote "glue" points in the patterns (later)
- Every node's degree must match the operator's arity:

$$\# \,\mathrm{op}\, v = \deg v$$

**Definition (Program Graph)**

A graph $G$ is a program graph if it is acyclic and

$$\forall v \in V : \mathrm{op}\, v \neq \square$$

# Patterns

- A graph $P = (V, E)$ is rooted if there exists a node $v \in V_P$ such that there is a path from $v$ to every node $v'$ in $P$
- If $P$ is rooted, denote the root by $\operatorname{rt} P$

**Definition (Pattern Graph, Pattern)**

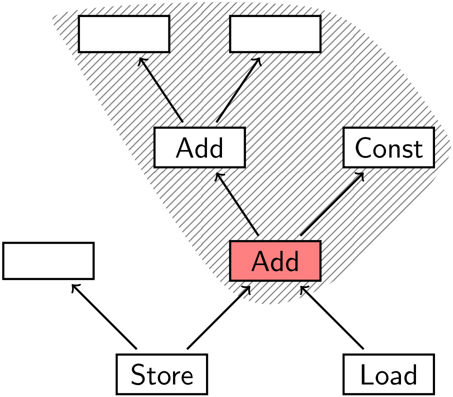
A graph $P$ is a pattern if

- it is acyclic and rooted
- $\operatorname{op} \operatorname{rt} P \neq \square$

- Note that we explicitly allow nodes with operator $\square$ in patterns

# Equivalence of Nodes in Patterns

- Complex patterns often have common sub-patterns




- Shall be treated as equivalent

- Selecting the common sub-pattern at the Add node shall enable selecting the complex instruction at Store and Load

# Equivalence of Nodes in Patterns

## Definition (Equivalence of nodes)

Consider two patterns $P$ and $Q$ and two nodes $v \in P$, $w \in Q$:

$$\begin{aligned} v \sim w :\iff\ & v = w \\ & \vee (\operatorname{span} v \cong \operatorname{span} w \wedge \operatorname{rt} P \neq v \wedge \operatorname{rt} Q \neq w) \end{aligned}$$

- Either the two nodes are identical
- $v, w$ are no pattern roots and their spanned subgraphs are isomorphic
- span $v$: induced subgraph that contains all nodes reachable from $v$

# Matching of a Node

- Let $\mathcal{P} = \{P_1, P_2, \dots\}$ be a set of patterns
- Let $G$ be some program graph

**Definition (Matching)**

A matching $\mathcal{M}_v$ of a node $v \in V_G$ with a set of patterns $\mathcal{P}$ is a family of pairs

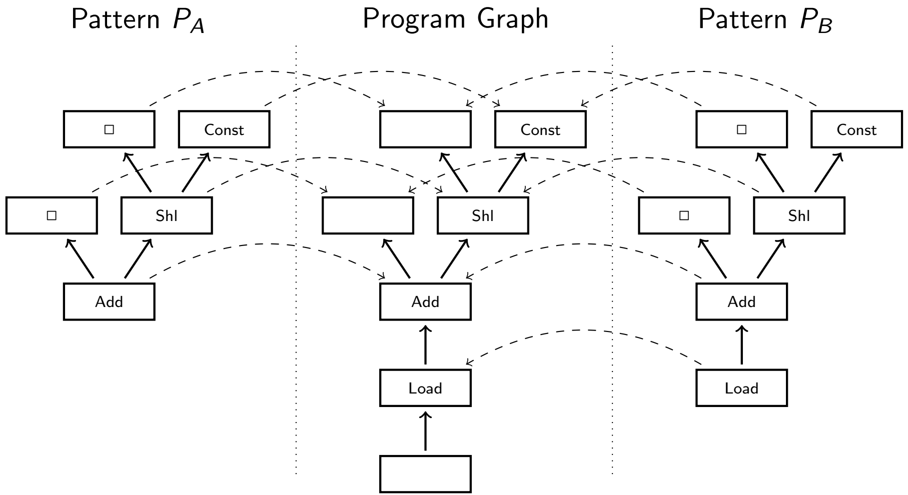
$$\mathcal{M}_v = \big((P_i, \imath_i)\big)_{i \in I} \qquad I \subseteq \{1, \dots, |\mathcal{P}|\}$$

of patterns and injective graph morphisms $\imath_i : P_i \to G$ satisfying

$$v \in \operatorname{ran} \imath_i \quad \text{and} \quad \operatorname{op} w \neq \square \implies \operatorname{op} w = \operatorname{op} \imath_i(w) \qquad \forall w \in P_i$$

# Matchings

## Example



# Selection

- We have computed a covering of the graph
- i.e. instruction selection possibilities
- Now, find a subset of the covering that leads to good and correct code
- Cast the problem as a mathematical optimization problem:

Partitioned Boolean Quadratic Programming (PBQP)

# PBQP

Let $\mathbb{R}_\infty = \mathbb{R}_+ \cup \{\infty\}$ and

- $\vec{c}_i \in \mathbb{R}_\infty^{k_i}$ be cost vectors
- $C_{ij} \in \mathbb{R}_\infty^{k_i} \times \mathbb{R}_\infty^{k_j}$ be cost matrices

## Definition (PBQP)

Minimize

$$\sum_{1 \le i < j \le n} \vec{x}_i^\top \cdot C_{ij} \cdot \vec{x}_j + \sum_{1 \le i \le n} \vec{x}_i^\top \cdot \vec{c}_i$$

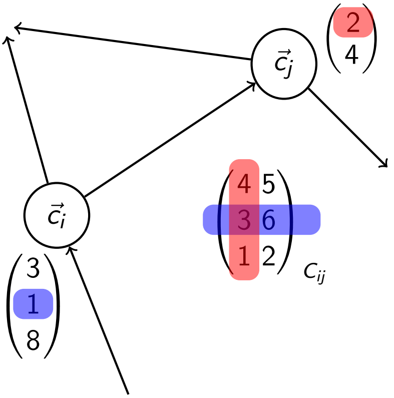
with respect to

$$\begin{aligned} &\vec{x}_i \in \{0,1\}^{k_i} \\ &\vec{x}_i^\top \cdot \vec{1} = 1, \quad 1 \le i \le n \\ &\vec{x}_i^\top \cdot C_{ij} \cdot \vec{x}_j < \infty, \quad 1 \le i < j \le n \end{aligned}$$

# PBQP

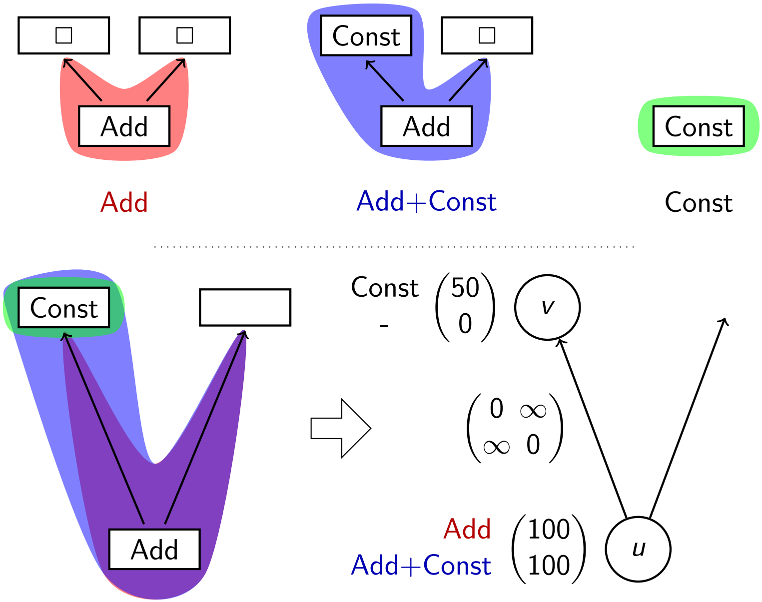
- $\vec{x}_i$ are selection vectors
- Exactly one component is 1
- This selects the component
- Cost matrices relate selection of made in different selection vectors
- Can be modelled as a graph:
  - cost vectors are nodes
  - matrices are edges
  - only draw non-null matrix edges

# PBQP as a Graph




- Colors indicate selection vectors $\vec{x}_i = (0\,1\,0)^\top$ and $\vec{x}_j = (1\,0)^\top$
- This selection contributes the cost of 6 to the global costs
- Edge direction solely to indicate order of $ij$ in the matrix subscript

# Mapping Instruction Selection to PBQP



# Mapping Instruction Selection to PBQP

Cost vectors are defined by node coverings:

- Let $\mathcal{M}_v$ be a node matching of $v$
- The alternatives of the node are given by partitioning the matchings by equivalence:

$$\mathcal{M}_{v/\sim}$$

- Common sub-patterns have to result in the same choice
- Costs come from an external specification

# Mapping Instruction Selection to PBQP

- Matrices have to maintain selection correctness
- Consider two alternatives

$$A_u = (P_u, \imath_u) \quad A_v = (P_v, \imath_v)$$

at two nodes $u, v$ connected by an edge $u \to v$.

- The matrix entry for those alternatives is

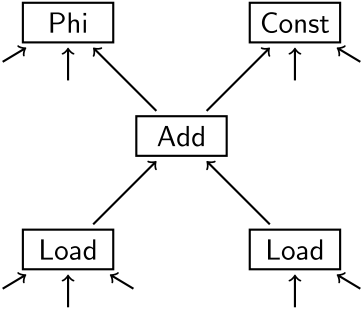
$$c(A_u, A_v) = \begin{cases} \infty & \text{op}\, \imath_u^{-1}(v) = \square \text{ and } \imath_v^{-1}(v) \neq \text{rt}\, P_v \\ \infty & \text{op}\, \imath_u^{-1}(v) \neq \square \text{ and } \imath_u^{-1}(v) \not\sim \imath_v^{-1}(v) \\ 0 & \text{else} \end{cases}$$

Id est:

- If $A_u$ selects a leaf at $v$, $A_v$ has to select a root
- If $A_u$ does not select a leaf, both subpatters have to be equivalent

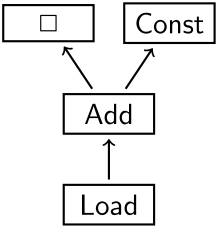
# Example

## Program Graph

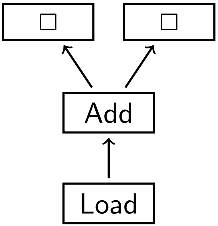


# Example

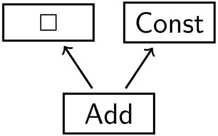
## Patterns




LAC (Load+Add+Const)



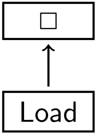

LA (Load+Add)



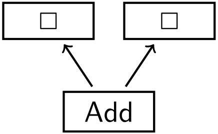

AC (Add+Const)


C (Const)


P (Phi)



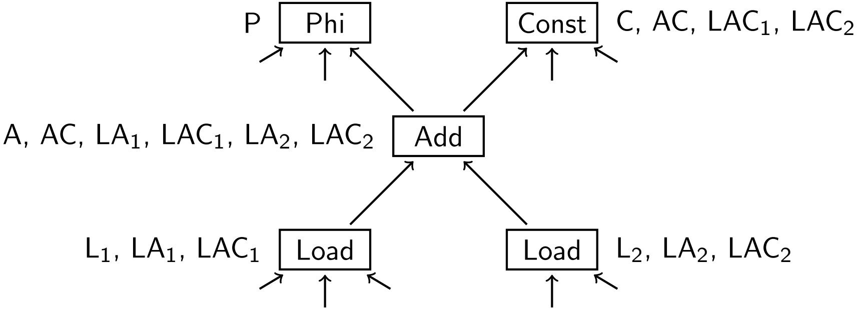

L (Load)




A (Add)

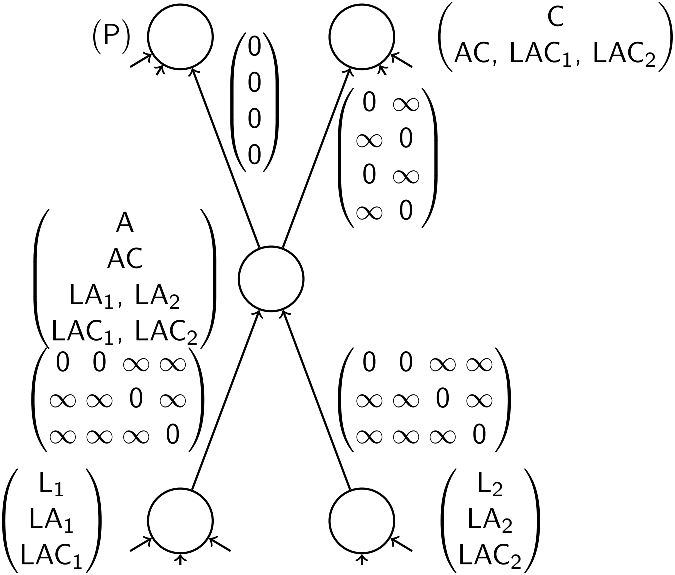
# Example

## Matchings



# Example

## PBQP Instance



# Reducing the Problem

Optimality-preserving reductions of the problem:

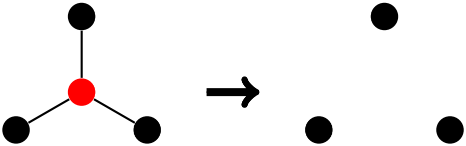
- Independent edges (e.g. matrix of zeroes):

- Nodes of degree 1:

- Nodes of degree 2:

# Reducing the Problem

- Heuristic Reduction:



  Chose the local minimum at a node

- Leads to a linear algorithm
- Each reduction eliminates at least one edge
- If all edges are reduced, minimizing nodes separately is easy

# Summary

- Map instruction selection to an optimization problem
- SSA graphs are sparse $\Longrightarrow$ reductions often applied
- In practice: heuristic reduction rarely happens
- Efficiently solvable
- Convenient mechanism:
    - Implementor specifies patterns and costs
    - maps each pattern to an machine node
    - Rest is automatic
- Criteria for pattern sets that allow for correct selections in every program not discussed here!

# Literature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